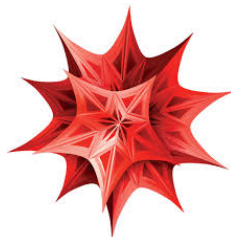Online Calculus II - Online Math 1220
Mathematica Commands: From Basics To Calculus II - Version 13©

Edit your document (remove extras and errors, ensure the rest works correctly) and upload your final Mathematica lab, .nb file. (Do not upload a pdf file. Upload the .nb file.) It is not necessary to respond to questions inside the parentheses, but you should answer them for yourself.

1. You may access the required software, called Mathematica, in the following three ways. 1. Get your own free copy by filling out the form at http://goo.gl/forms/tUypRLjwhS . 2. Any campus computer lab, including Tracy Hall Computer Lab, TY 126, and Elizabeth Hall Computer Lab, EH 213. 3. Virtual Lab: For the Virtual Lab instructions, see http://www.weber.edu/VirtualLab/GettingStarted.html .

2. Here is how to get on Mathematica:

   Log in on a computer. Move the mouse pointer to the **Mathematica icon**, which looks like the above star like symbol, and rapidly **click twice** using the **left mouse button**. You may find the mathematica icon on the desktop, under the button Start - Programs - Wolfram or Mathematica, or by doing a search for Mathematica. Then click on NewDocument , or on New Notebook, if a notebook doesn't automatically open. If you need help, ask the lab aide.

3. Entering information is like using a word processor. You can also use the arrow and backspace keys for editing. Try typing some text and practice editing it. Mathematica displays all work in "cells". The brackets on the far right indicate the cell. You can create a new cell for your next input by clicking the mouse below the cell you have been working in. First a horizontal line will appear and then when you type a new cell will be created.

   **Throughout this lab and future labs input the statements in** `True Type` **font exactly as is!**

4. Mathematica will do arithmetic operations using the usual symbols $+, -, *, /, \hat{}$ . A blank space between two terms also indicates multiplication but using $*$ might facilitate later editing. To see how it works, enter the following calculation followed by the appropriate **Evaluation** (Execution) key command; the **Enter** button on the number pad (not $\boxed{\longleftarrow}$ on the main keyboard) should work on all computers.

$$4*8 \quad \boxed{\textbf{Enter}} \text{ (in the number pad, not the Return key)}$$

(In place of the $\boxed{\textbf{Enter}}$ key, you may also use $\boxed{\text{Shift}}\boxed{\longleftarrow}$ .) Now use Mathematica to evaluate the following. Remember to push the $\boxed{\textbf{Enter}}$ key after each part. Check the answers using a calculator. If the output is the same as the input, you probably have made an error in typing.

   (a) `3+6*5`  (Does Mathematica follow order of operation rules?)
   (b) `274/3`  (Try this again with a decimal point after either number.)
   (c) `((3.24*6.791)-14.7)/(4.5+82/3)`  (Mathematica will not accept brackets or braces as computational grouping symbols. Try $[(3.24 * 6.791) - 14.7]/(4.5 + 82/3)$ . Do you get the same answer?)
   (d) `2.6^2.99`  (The symbol $\hat{}$ is the power sign.)
   (e) `3 5`  (Don't forget the space between 3 and 5.)

5. All built-in Mathematica commands or words begin with a capital letter and use "brackets" if a grouping symbol is appropriate. Type in and evaluate each of the following.

   (a) `Log[12.7]`   (This is the natural logarithm $\ln 12.7$.)

   (b) `E^2.5`   (This is $e^{2.5}$.)

   (c) `Log[10,72.8]`   (This is the common logarithm $\log 72.8$. Notice that the first number indicates the base.

   (d) `Sqrt[64.0]`   (This is $\sqrt{64}$.)

   (e) `Abs[-2.5]`   (This is $|-2.5|$.)

   (f) `Sin[Pi/6]`   (This is $\sin(\pi/6)$. Trigonometric functions are defined as a function of radians. If a degree unit is used, it must be indicated as shown next.)

   (g) `Cos[30 Degree]`   (This is $\cos(30°)$.)

   (h) `ArcTan[1]`   (This is $\tan^{-1}(1)$.)

   (i) `I*I`   ($I$ is $i = \sqrt{-1}$.)

6. You can find a numerical approximation using the function `N` and `NumberForm`. Type in and evaluate each of the following.

   (a) `E`   (This symbol stands for the exact value of $e$ and will only be replaced with its numerical value if asked.)

   (b) `N[E]`   (Approximate value of $e$.)

   (c) `Pi`   (This symbol stands for the exact value of $\pi$ and will only be replaced with its numerical value if asked.)

   (d) `N[Pi]`   (Approximate value of $\pi$.)

   (e) `Pi//N`   (This is another way of asking for the numerical value of the $\pi$.)

   The function `N` calculates the answer to 16 significant figures (not decimal places) but *Mathematica* only displays the default number of places. For viewing the answer to more than the default number of display places but **no more than 16** significant figures you must use the `NumberForm` command along with `N`. Try the following.

   (f) `NumberForm[N[E],10]`

   (g) `NumberForm[N[Pi], 16]`

   You can use the command `N` by itself for calculating and displaying the answer for **more than 17** significant figures. Try the following.

   (h) `N[Pi, 17]`   (Did you get one more digit than the last part?)

   (i) `N[Sqrt[2], 50]`   (This gives the value of the irrational number $\sqrt{2}$ to 50 places.)

7. You can also input mathematics in a more standard form using palettes. A basic palette is usually present on the top right hand side of the screen. (If it is not, you can get it by clicking consecutively on the buttons Palettes, and Basic Math Assistant.) You can get a form by clicking on it in the palette. You can fill a form by clicking on each location and typing. You can also **Tab** between different locations in a form. Use the palette to type in each of the following and then evaluate each one.

   (a) $3^{5.2}$

   (b) $\sqrt{37.4}$

   (c) $\sqrt{3.1^{2.7}}$

   (d) $\frac{125.3}{72}$

8. The % key allows you to call back a previous output line. To call back the output from the previous line enter %, two lines up %%, 3 lines up %%%, etc. Evaluate the following.

   (a) `N[E^2]`

   (b) `Log[%]`    (This is the natural logarithm of the last output or $\ln(e^2)$ which, of course, is equal to 2.)

   (c) `Log[%%]`    (This is the natural logarithm of the two outputs before, which is again $\ln(e^2) = 2$.)

   (d) `Log[%]`    (What is this? Verify your answer!)

9. Mathematica can work symbolically to perform algebraic operations. Try these.

   (a) `Expand[(a+1)^5]`

   (b) `Simplify[3x^2-x-9+x^2+7x+5]`

   (c) `Factor[x^4 - 1]`

   (d) `Together[2/(x-3)-1/(x+2)]`

   (e) `Apart[5/((x-3)(x+2))]`    (The partial fractions technique command is `Apart`.)

   (f) `Cancel[(x^2-2x-3)/(x^2-9)]`

10. Replacement. A single equal sign indicates replacements. Type in the following and evaluate them.

    `x = 5; y = 12; z = a + b;`

    Enter all of them in just one cell; you may enter each on a separate line or all on one line. If you type in more than one on the same line, be sure to separate them by semicolons. The final semicolon in a line suppresses the final output. If you have used all the semicolons, you should not get any output! Now evaluate the following.

    (a) `x`    (This should give the assigned value of $x$ which is $5$.)

    (b) `z`    (This should give the expression that $z$ stands for: $a + b$.)

    (c) `x y^2`    (Remember to insert the space between $x$ and $y$ to indicate a product.)

    (d) `Expand[z^x]`    (This is $(a + b)^5$.)

    The assignments for $x$, $y$ and $z$ are permanent; this means that to reuse these variables for something else requires that they first be "cleared". Type in the following and evaluate it.

    (e) `Clear[x,y,z]`

    (f) `x`    (You should get $x$ and not its previous value of 5. It is a good idea to clear each variable after its use, otherwise its reuse will give unintended results.)

11. In Mathematica, an equation is denoted by using two equal signs. For example, the equation `3x+2==x-5` represents the equation $3x + 2 = x - 5$. You can find exact solutions of many equations using the command `Solve`. Type in and evaluate each of the following. The solution is given as a replacement value for the variable ($\{x \to \texttt{answer}\}$).

   (a) `Solve[3x+2==x-5]`    (If the output is an empty set, either this equation has no solution or you have forgotten to clear the value of $x$ earlier. Check you answer.)

   (b) `Solve[z^2+2z==-2]`   (Did you clear $z$ earlier? What type of numbers are the solutions? When does a quadratic equation have two complex conjugate solutions?)

   (c) `Solve[a t-2==3a, t]`   (The $t$ after the comma indicates that the equation should be solved for $t$ and not $a$. If you want to simplify your answer, use `Simplify[%]`.)

   (d) `Solve[{x-2y==4, x-1==5y}, {x, y}]`   (To solve 2 or more equations, list all equations and unknown variables using braces, as shown.)

   If an equation cannot be solved exactly or an exact solution is not desired, then a numerical solution may work. Below are two such examples.

   (e) `NSolve[{x^2+y^2==16,y==x^2-2x+2},{x,y}]`   (The solutions include points of intersections of the circle $x^2 + y^2 = 16$ and the parabola $y = x^2 - 2x + 2$.)

   (f) `FindRoot[Cos[x]==x^3,{x,1}]`   (This command applies the Newton's method to the equation $\cos x = x^3$ with the starting value $x_1 = 1$.)

12. The equal sign for entering functions is `:=` (not just `=`). Its independent variable is indicated by an underline ($\boxed{\text{shift}}$ $\boxed{-}$) after it. Both are placed in one bracket pair. Do not use capital letters in naming functions since it might get mixed up with Mathematica's reserved functions. Enter the following functions and then perform the indicated steps.

   `f[x_]:=x^2 - 5; g[x_]:=Sin[x]; h[x_]=ArcSin[x]`   (The functions defined are $f(x) = x^2 - 5$, $g(x) = \sin x$ and $h(x) = \sin^{-1} x$ and the independent variable for all is $x$. Here, a semicolon at the end of the line has no effect!)

   (a) `f[3]`

   (b) `f[g[Pi/6]]`   (This is $f(g(\pi/6))$.)

   (c) `Solve[f[x]==4, x]`   (This solves the equation $x^2 - 4 = 5$.)

   (d) `h[g[3 Pi/4]]`   (This is $\sin^{-1}(\sin \frac{3\pi}{4})$. Is the answer correct!)

13. You can draw a function, relation, or equation using the `Plot` command. Each expression must be followed by a list which gives the variable(s) and the domain (minimum value to maximum value). Enter the following.

   (a) `Plot[(x-1)/(x^2+4),{x, -5, 6}]`   (The function graphed is $y = \frac{x-1}{x^2+4}$ and the domain is $-5 \le x \le 6$.)

   (b) `Plot[f[x],{x, -4, 4}]`   (The gives the graph of $f(x) = x^2 - 5$, which you inputted above, on the domain $[-4, 4]$.)

   (c) `Plot[2Sin[x+Pi/4] - Cos[x],{x,-4Pi, 4Pi}]`

13. (Continued.)

  (d) You can graph a polar equation using the `PolarPlot` command.
      `PolarPlot[Sin[2t],{t,0,2Pi}]`   (This is the graph of $r = \sin(2\theta)$. I used $t$ in place of $\theta$ since it was easier to type in!)

  (e) To graph two or more functions on the same coordinate system, enter them as a list.
      `Plot[{Cos[x],x^3},{x,-2Pi,2Pi}]`   (This is the graph of the two functions $y = \cos x$ and $y = x^3$ on the same coordinate system.)

  (f) You can read the coordinates of a point on a graph by first right clicking on the graph and choosing Get Coordinates (or, first, Drawing Tools) in the window that opens up. Move the cursor over the graph. The coordinates will appear on the screen. Now, try to read the coordinates of the point of intersection of the two curves in the last part. (Is the $x$ value of the point of intersection approximately what you got from the `FindRoot` command in the earlier part of this lab, or about 0.86?)

14. We can use Mathematica to evaluate limits. The `Limit` command of Mathematica has been improved, but one must still verify that the limit exists graphically or numerically. Enter the following.

  (a) `Limit[Sin[x]/x,x->0, Direction->`"FromBelow"`]`   (This is $\lim\limits_{x \to 0^-} \frac{\sin x}{x}$.)

  (b) `Limit[Sin[x]/x,x->0, Direction->`"FromAbove"`]`   (This is $\lim\limits_{x \to 0^+} \frac{\sin x}{x}$.)

  (c) `Limit[Sin[x]/x,x->0]`   (Since the left and right limits are equal, this is the correct value of $\lim\limits_{x \to 0} \frac{\sin x}{x}$.)

  (d) To investigate $\lim\limits_{x \to 0} \frac{|x|}{x}$, first graph this function near $x = 0$.
      `Plot[Abs[x]/x, {x,-1,1}]`   From this graph it is clear that the left and right limits as $x \to 0$ are not equal, and so, the limit does not exist.
      `Limit[Abs[x]/x,x->0]`   Mathematica can not determine a limit value, and you know the reason is that the limit does not exist.

  (e) You can evaluate $\lim\limits_{x \to \infty} \frac{\ln x}{\sqrt{x}}$ by l'Hospital's rule. Mathematica can also evaluate it.
      `Limit[Log[x]/Sqrt[x],x->Infinity]`

15. Mathematica can do symbolic differentiation.

  (a) `D[x^2Sin[x]-3x+1,x]`   (This gives the derivative of $y = x^2 \sin x - 3x + 1$ with respect to $x$.)

  (b) You can also use the prime notation.
      `f[x_]:=x^3-2x^2+5`
      `f'[x]`   (Is $f'(x) = 3x^2 - 4x$?)
      `f''[x]`   (This gives the second derivative $f''(x)$.)
      `D[f[x],{x,2}]`   (This is another way of finding the second derivative $f''(x)$.)

  (c) You can also perform implicit differentiation.
      `D[x^5 y[x]^2 + x Sin[y[x]]^3==1,x]`   This differentiates the equation $x^5 y^2 + x \sin^3 y = 1$ with respect to $x$. (`y[x]` indicates $y$ is a function of $x$.)
      `Solve[%,y'[x]]`   (Now, you have solved for $y'$.)
      `D[x^5 y[x]^2 + x Sin[y[x]]^3==1,{x,2}]`   This differentiates the same equation with respect to $x$ twice.
      `Solve[%,y''[x]]`   (Now, you have solved for $y''$.)

16. Mathematica does symbolic integration. Mathematica evaluates definite integrals both numerically, and, also, by doing symbolic integration first. Input the following.

   (a) `Integrate[x^2+1,x]`   (Mathematica gives the antiderivative of $x^2 + 1$ with $C = 0$.)

   (b) `Integrate[E^x Sin[x],x]`   (This is $\int e^x \sin x \, dx$ with $C = 0$.)

   (c) `Integrate[x^2+1,{x,-1,2}]`   (This evaluates $\int_{-1}^{2} (x^2 + 1) \, dx$ by first finding the antiderivative.)

   (d) Numerical Integration. Often we do not know the antiderivative of a function but we can approximate its definite integral numerically.

   `NIntegrate[E^(-x^2),{x,0,Infinity}]`   (This evaluates $\int_{0}^{\infty} e^{-x^2} \, dx$ numerically.)

   (e) However, you can hand evaluate some integrals more efficiently than Mathematica. The easy way to evaluate $\int 3x^2(2010 - x^3)^{1999} \, dx$ is by u-substitution. Mathematica expands its integrand before evaluating it.

   `Integrate[3x^2 (2010-x^3)^1999, x]`

17. The `Series` command can be used to find the power series expansion of a function about a given point up to a given desired power. The `Sum` command can find the value of some sseries.

   (a) `Series[E^x, {x,0,6}]`   (This is the Maclaurin series of $f(x) = e^x$ with all terms up to the sixth power stated explicitly and `O[x]`$^7$ stands for all higher order terms.)

   (b) By adding the command `Normal` a series is truncated to a polynomial.
   `Normal[Series[Cos[x], {x,0,8}]]`   (This is the 8th-degree Maclaurin polynomial of $f(x) = \cos x$.)

   (c) `Series[Log[x], {x,1,5}]`   (This is the Taylor series expansion of $f(x) = \ln x$ about $x = 1$ with all term up to the fifth power stated explicitly.)

   (d) Mathematica can find the sum of certain series.

   `Sum[1/n^2, {n,1,Infinity}]`   (This gives the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.)

18. If you are using a university computer lab, please leave the computers as you find them. When you are done, close the software being used and log off properly.